Finite Element Modeling of FR Concrete Beam

Wadhah M. Tawfeeq[1], Hazim F. El-Erris[2]

Abstract— The use of steel fibers in plain and reinforced concrete demonstrates a number of advantages, such as increasing the flexural capacity, stiffness, ductility and energy absorption capacity. Under the service load, there is an improvement in crack control characteristics. In the present study, the behavior of steel fibers reinforced concrete beams due to the external applied load is studied by using finite element method. A non-linear finite element computer program to simulate the behavior of FR concrete beams under the action loading has been developed. Eight nodded two-dimensional isoparametric elements are used to simulate the fibrous concrete. The smeared representation is used to model the reinforcement within the concrete. The compressive strength of concrete is simulated by an elasto-plastic model followed by a perfectly plastic behavior, which is terminated at the onset of concrete crushing. A smeared crack model with fixed orthogonal cracks is used to simulate the concrete response in tension. Tension stiffening concept is used to simulate the post-cracking behavior of concrete. The capabilities of the program have been examined and demonstrated by analyzing experimental problem of fibrous reinforced concrete beams. The accuracy of the numerical results is assessed by comparison with the experimental results, which are shown to be in good agreement.

Index Terms— Nonlinear, Finite Element, Steel Fiber, RCC, Tension Stiffening, Load-Defliction, Iisoparametric Element.

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1 Introduction

SINCE the early fifties of the last century, matrix-algebra has been applied to structural analysis. During this period the flexibility and stiffness methods are proposed as an outgrowth of classical methods for the analysis of one-dimensional problems. These matrix methods were expanded in 1956[1], into the finite element method, for the analysis of two-dimensional problems. Until the late 1950's, most methods of structural analysis were based on linear elastic behavior leading to working-stress design method. Non-linear material behavior is necessary when structures are analyzed on the basis of their ultimate strength. This will enable engineers to predict more accurately not only the overall strength of structures, but also their behavior at different load levels. Since the late 1960's, the finite element method has been used extensively for the non-linear analysis of reinforced concrete members such as beams and slabs. Although the behavior of RC members is extensively investigated, little work has been devoted to study the behavior of fibrous reinforced concrete members.

Al-Ta'an[2], used the finite element method for the non-linear analysis of fibrous reinforced concrete beams. The beams were assumed in a state of plane stress. The concrete and steel reinforcement was represented by constant strain triangular element. Non-linearities due to cracking, compressive behaviour of concrete, steel and bond-slip were taken into account. The ultimate loads, deflection, crack propagation and crack patterns are predicted satisfactorily.

Al-Ta'an and Ezzadeen[3] adopted a numerical procedure based on the finite element method for the non-linear analysis of fibrous reinforced concrete members subjected to monotonic loads. The proposed method was capable of tracing the response of these structures up to failure. The predicted results include displacements, strain, curvature, slopes and stresses. The iterative scheme based on Newton-Raphson method has been employed for non-linear solution algorithm. A beam element with composite layer system was used to model the structure.

Allose[4], investigated the behavior of reinforced fibrous concrete beams subjected to torsion using non-linear-three dimensional finite element model. Eight- and twenty noded isoparametric brick elements were used to model the concrete. the reinforcing bars were idealized as axial members embedded within the concrete element and perfect bond between the concrete and the reinforcement was assumed. The compressive behaviour of the steel fibers reinforced concrete was simulated by an elasto-plastic work hardening model followed by perfectly plastic response, which was terminated at, the onset of crushing. In tension, a smeared crack model with fixed orthogonal cracks was used with the inclusion of models to account for the retained post-cracking stress and the reduced shear modulus. The non-linear equations of equilibrium were solved using an incremental-iterative technique operating under load control. The solution algorithms used were the standard and the modified Newton-Raphson methods. The numerical integration was generally carried out using the 15-points Gaussian type rules.

Ayad[5] proposed a procedure for non-linear analysis of reinforced fibrous concrete members. A degenerated quadratic thick shell 8-noded element was used to model the concrete. Two approaches were used in the analysis of beams; the plane stress and the layered approaches. The layered approach was usually used for analysis of slabs. Two representations were implemented for the steel reinforcement, the smeared representation, which was incorporated with the layered approach, and the embedded representation, which was incorporated with the plane stress approach. Perfect bond at the interface of concrete and reinforcement was assumed. Both an elastic-perfectly plastic and strain hardening plasticity approaches were used to model the compressive behaviour of steel fibers reinforced concrete. The steel reinforcement was considered as an elastic perfectly plastic material or as an elastic-plastic material with strain hardening. The computed results of various examples including shallow and deep beams, corbels, slabs

- *Wadhah M .Tawfeeq[1] is currently an Assistant Professor, Faculty of Engineering, Sohar University, Sohar, PC 311, Oman; email: WTawfeeq@soharuni.edu.om or wadhah78@yahoo.com*

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and shells were compared with published experimental results.

Naji[6] extended his work[7] to take into account the addition of steel fibers to reinforced concrete beams. The modification included the tension stiffening model which was used to model the behavior of steel fibers reinforced concrete in tension and calculation of yield function and flow function used to simulate the behaviour of steel fibers reinforced concrete in compression.

2 Finite Element Representation

Isoparametric elements have enjoyed considerable appeal for their simplicity and versatility. Among the most widely used, is the eight-nodded isoparametric element, shown in Figure 1. It has 16-degrees of freedom with two translation displacements at each node. The element is capable of accommodating curved or non-rectangular boundaries. This element has been adopted in this work.

The isoparametric element family derives its name from the fact that the same interpolation function, used to define the element shape based on the nodal coordinates, is also used to define the element displacements based on the nodal displacement values.

A local non-dimensional coordinates system is used to define the interpolation function. It is called the natural coordinates system because the coordinate values are always in the range of ± 1.0. However, the natural coordinates axes ξ and η pass through the midpoints of apposite sides such that the element sides are defined by $\xi = \pm 1.0$ and $\eta = \pm 1.0$.

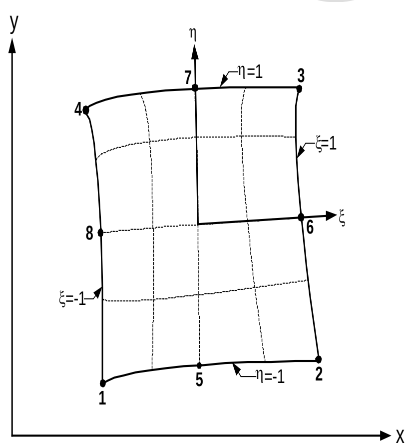

Fig.1. Typical 8-noded isoparametric element.

3 Tension Behavior And Cracking Models

In tension, concrete is assumed to be linearly elastic up to cracking. The onset of cracking is predicted by the maximum tensile stress criterion defined in terms of principal stresses σ_1 and σ_2. Cracks can be modeled either as discrete individual cracks between concrete elements or as smeared cracks within the elements. The smeared cracks representation can be divided into fixed and rotating crack categories. The represented cracks follow the fixed orthogonal smeared crack approach. After cracking the stress normal to the crack is gradually released according to tension-stiffening models related to the fracture energy. Partial and full closings of cracks are permitted and the degradation in compressive strength of concrete normal to crack opening is also taken into account.

♦ TS2 Model

Many researches [7,8,9,10] used the tension-stiffening model, as shown in Figure 2. Furthermore, this model has been adopted in the present work. This model is given by:

a) For $\varepsilon_{cr} \le \varepsilon_n \le \alpha_1 \cdot \varepsilon_{cr}$

$$\sigma_n = \alpha_2 \cdot \sigma_{cr} \cdot \left[\frac{\alpha_1 - \dfrac{\varepsilon_n}{\varepsilon_{cr}}}{\alpha_1 - 1.0} \right]$$

b) For $\varepsilon_n > \alpha_1 \cdot \varepsilon_{cr}$

$$\sigma_n = 0.0$$

where σ_n and ε_n are the stress and strain normal to the cracked plane, ε_{cr} is the cracking strain associated with the cracking stress σ_{cr} and α_1 and α_2 are the tension-stiffening parameters. α_1 represents the rate of stress release as the crack widens, α_2 represents the sudden loss of stress at instant of cracking.

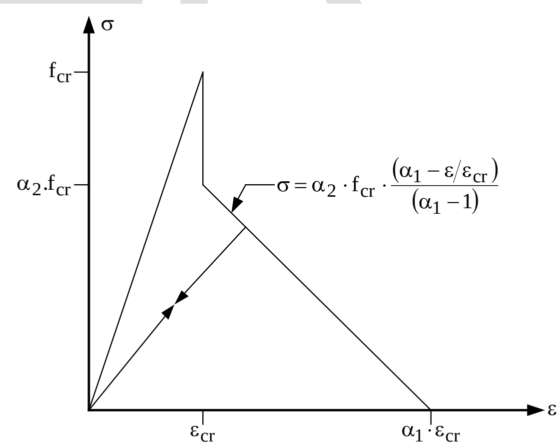

Fig.2. Pre and post-cracking behaviour of SFRC TS2 model. Ref. (9).

4 Modeling Of Reinforcement

In the present research work, steel reinforcement is smeared into equivalent steel layer with uniaxial properties following the bar direction. In contrast to concrete, the mechanical properties of steel reinforcement are well known. The uniaxial stress-strain relation for steel is idealized as bilinear curve, representing elasto-plastic behavior with strain hardening.

This relation is assumed to be identical in tension and compression, as shown in Figure 3.

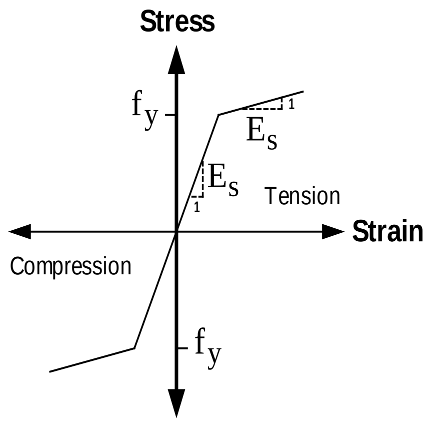

Fig.3. Stress-strain relationship for steel. Ref. (10).

5 Computer Program

A computer program has been developed to carry out the analysis of the two-dimensional fibrous reinforced concrete beams subjected to loading. The program is primarily based on a program developed by Al-Sa'aedi[10] for the non-linear finite element analysis of reinforced concrete beams. The program is coded in FORTRAN-77 language. Generally the material non-linearity is due to the plastic deformation, cracking and crushing of concrete, and yielding of reinforcing bars. The computer program includes all the material non-linear factors. It consists of a main program and fourty-three subroutines.

6 Application

In this section, the problem is analyzed using the computer program. The applied loads are incremented according to a specified load factor. The program is applied to the analysis of a simply supported beam. The results obtained show good correlation when compared with the experimental data.

6.1 Simply Supported SFRC Beam A1

A simply supported steel fibers reinforced concrete beam is analyzed using the capability of the present program in treating the shallow fibrous reinforced concrete beams.

6.1.1 Description of the Beam A1

The beam was tested experimentally by Alsayed[11], with two concentrated loads of 60 kN each located at third points of the beam, as shown in Figure 4. The beam is reinforced with two longitudinal bars each of 201mm^2 cross sectional area in the tension zone and one longitudinal bar of 79mm^2 cross sectional area in the compression zone.

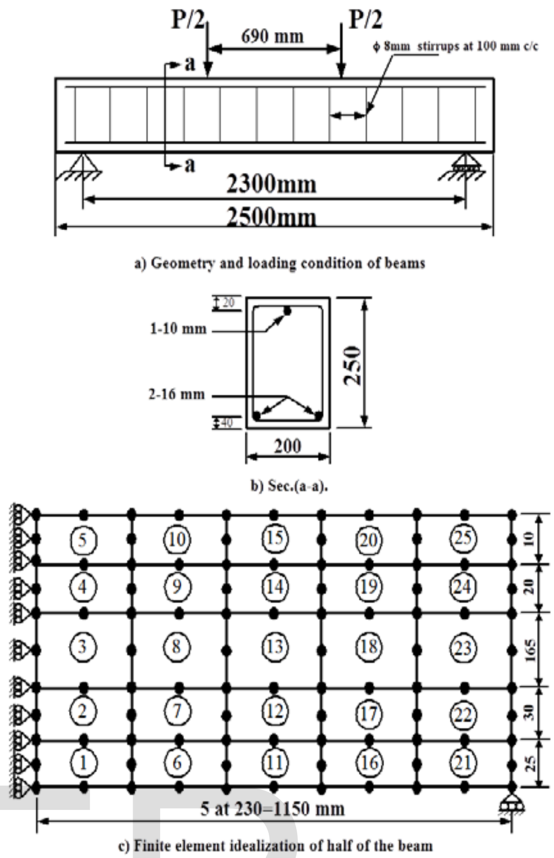

Fig.4. Simply Supported SFRC Beam.

6.1.2 Finite Element Idealization and Material Properties

The beam, shown in Figure 4-a, is analyzed by the finite element method using the proposed solution. Due to symmetry of loading and geometry, only one half of the beam is analyzed using 25 elements. The finite element mesh, boundary conditions and loading arrangement, used in the analysis, are shown in Figure 4. Material properties of concrete, steel and steel fibers are:

E_c=27000 MPa, υ_c =0.15, $f_{c'}$ =40 MPa, $f_{t'}$ =3.7 MPa, ε_{cu} =0.0035, Es=200000 MPa, f_y =470 MPa, As' =1$\times$79 mm^2, As=2$\times$201 mm^2, l_f =60 mm, d_f =1.0 mm, V_f =1.0%.

Where E_c is the concrete Young's modulus, υ_c is the concrete Poisson's ratio, $f_{c'}$ is the concrete compressive strength, $f_{t'}$ is the concrete tensile strength, ε_{cu} is the ultimate strain of concrete in compression, E_s is the steel Young's modulus, f_y is the steel yield stress, As' is the area of steel in the compression zone, As is the area of steel in the tension zone, l_f is the length of steel fibers, d_f is the diameter of steel fibers, and V_f is the volume fraction.

6.1.3 Effect of Volume Fraction

Values of volume fraction equal to (0.0%, 0.5%, 1.0%, 1.5%) have been adopted, see Figure 5. In this figure, the effect of volume fraction on the load-deflection curve behavior and the advantages of using steel fibers in reinforced concrete beams can be seen. For TS2 model; see Figure 5; it can be seen that there is no difference between the results when the volume fraction is changed because the volume fraction is used for calculating the material properties only and does not exist in the tension stiffening model; it can be noticed that the increase in volume fraction is followed by a slight decrease in the deflections.

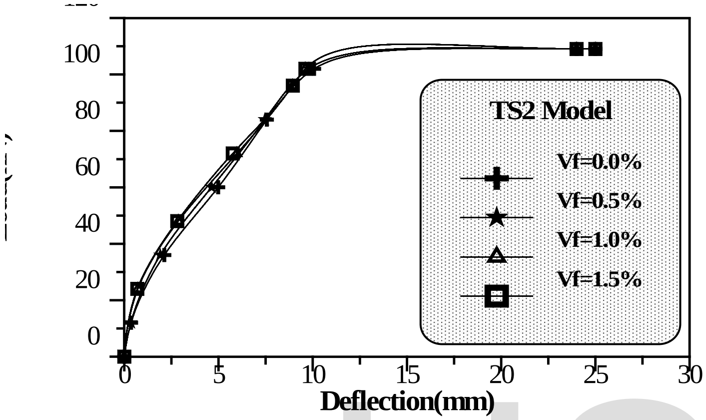

Fig.5. Load-deflection curves at mid-span of the beam affected by volume fraction of fibers.

6.1.4 Effect of Steel Fibers Diameter

Values of steel fibers diameter are equal to (0.8, 1.0 and 1.5 mm). In Figure 6, it can be seen that steel fibers diameter has insignificant effect on the load-deflection curve of the beams. In general, the increase in steel fibers diameter is followed by a decrease in deflections, which indicates that the steel fibers arrest the cracks from expanding, therefore, increase the tensile strength of the composite.

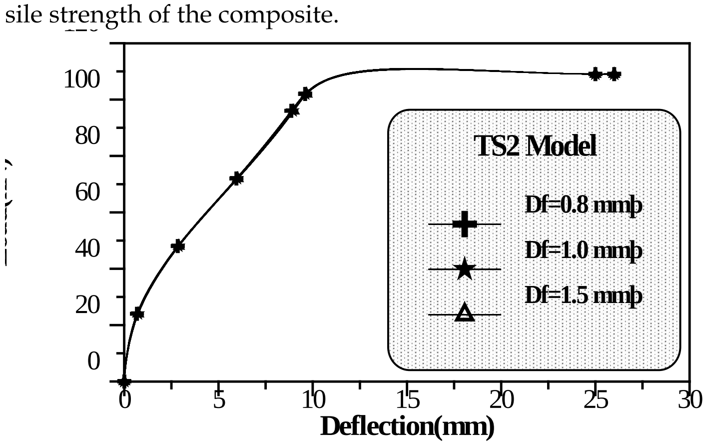

Fig.6. Load-deflection curves at mid-span of the beam affected by diameter of fibers.

6.1.5 Effect of Steel Fibers Length

Values of the steel fibers length are equal to (50,60 and 65 mm). In Figure 7, it can be noticed that the effect of the steel fiber length is insignificant and the increase of steel fibers length is not followed by any decrease in the deflections.

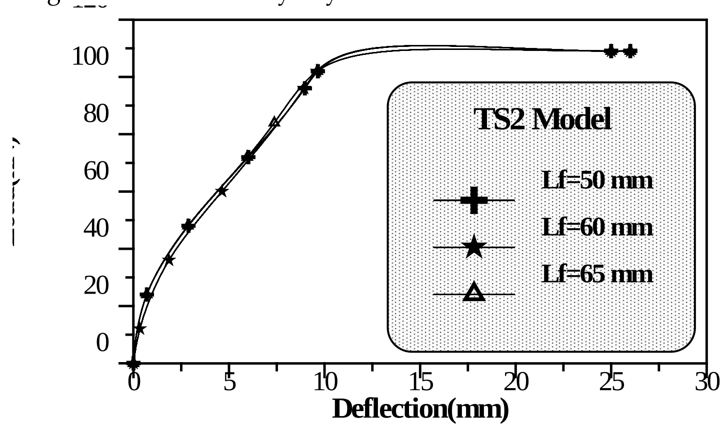

Fig.7. Load-deflection curves at mid-span of the beam affected by length of fibers..

6.1.6 Results of the Analysis

Alsayed[11], as stated previously obtained the experimental results of this beam. His results showed that fiber addition to the concrete has an insignificant effect on the modulus of elasticity of the plain concrete. It was also shown that fibers are the most effective in reducing deflections after the section has cracked.

In the present study, TS2 Model is used to represent the post-cracking stresses because the effect of steel fibers is more important in calculating the post-cracking model. The main reasons for using steel fibers in reinforced concrete beams are arresting crack growth and crack propagation and to increase the tensile strength of concrete. The TS2 model gives deflection close to the experimental deflections for all stages of loading.

7 Conclusion

Depending on the numerical results, obtained from the finite element solution, carried out throughout the present research, the following conclusions can be made:

- The present method of analysis has proved to be capable of predicting the non-linear response of fibrous reinforced concrete beams**.** The two dimensional finite element analyses, adopted in the present study, may be considered as an effective method commensurate with the available memory for the computers**.**
- The smeared cracking model, used to describe the cracking of the fibrous reinforced concrete beams, analyzed in this study, is seen to be acceptable**.**
- Addition of steel fibers to high-strength concrete beams reduces immediate as well as total long-term (immediate plus time-dependent) deflections**.**
- The values of fiber volume and the amount of compression reinforcement affect on the inclusion of steel fibers and compression.
- The dimension of the steel fibers has insignificant effect on the load-deflection behavior of fibrous reinforced concrete

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beam.

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